On Measurements of Microwave $\overline{E}$ and $\overline{H}$ Field Distributions by Using Modulated Scattering Methods*

MING-KUEI HU†

Summary—The modulated scattering method of Justice, Rumsey, and Richmond for measuring $\overline{E}$ field distribution is extended to the measurement of $\overline{H}$ field distribution by using a loop scatterer formed by two diodes. This diode loop method has the particular advantage of eliminating the large and undesirable effect produced by the associated $\overline{E}$ field when measuring the $\overline{H}$ field.

A scattering analysis of the modulated diode loop is presented. It explains the principle of this new method and also supports the advantage mentioned above. A similar analysis for the modulated diode scatterer used in measuring $\overline{E}$ is also presented. It is believed that the explanation based upon this analysis for the $\overline{E}$ measurement is more satisfactory than that given by Richmond which is based upon a qualitative description of the diode scatterer.

A THEORETICAL basis and some experimental results on an ingenious scattering method for measuring $\overline{E}$ field were published by Justice and Rumsey.[1] The method was later modified by Richmond[2] with the use of a modulated diode as the scatterer. This modulated scattering method has been extended by the present author to the measurement of $\overline{H}$ field by using a loop scatterer formed by two diodes. This diode loop method has the particular advantage of eliminating the large and undesirable effect produced by the associated $\overline{E}$ field when measuring the $\overline{H}$ field. Such $\overline{E}$ field effect is really what makes most of the $\overline{H}$ field measurements unreliable.

A scattering analysis of the modulated diode loop has been worked out. It explains the working principle of this new method for measuring $\overline{H}$ and also supports the advantage mentioned above. A similar analysis has also been applied to the modulated diode scatterer used by Richmond. An explanation, based upon this analysis, for the working principle of the modulated scattering method for measuring $\overline{E}$ is believed to be more satisfactory and enlightening. For the convenience of presentation, the simpler analysis of a modulated diode is given in Section I. The analysis of a modulated diode loop and its application to the measurement of $\overline{H}$ field are presented in Section II.

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† Dept. of Elec. Engrg., Syracuse University, Syracuse, N. Y.

[1] R. Justice and V. H. Rumsey, "Measurement of electric field distributions," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-3, pp. 177–180; October, 1955.

[2] J. H. Richmond, "Measurement of field distributions," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 13–15; July, 1955.

I. Back-Scattered Signal From a Diode Dipole and $\overline{E}$ Measurement

The method of analysis is an extension of that used by Y. Y. Hu in a paper on back-scattering cross section.[3] In Fig. 1, with the effect of the slightly conducting leads for modulation neglected, the diode with its two short conducting leads is considered as a short dipole center loaded with the diode junction impedance. The two terminals across the source which produces the field to be measured and the two terminals across the diode junction are considered as a two-port configuration. The two pairs of terminals of this two-port will be referred to as the source terminals and the dipole terminals in the following analysis.

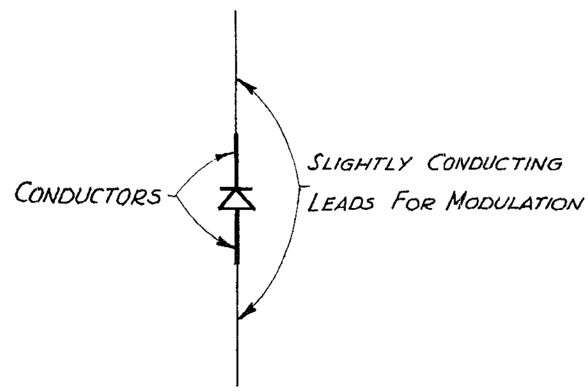

Fig. 1—A diode dipole scatterer.

It is clear that, with the diode junction impedance removed, the remaining part is linear; therefore, the principle of superposition applies. As far as the junction impedance is concerned, it serves only as a link between the dipole terminal current and the dipole terminal voltage, and the modulation merely changes the value of this junction impedance. Based upon such an approach, the back-scattered signal at the source terminals due to a loaded dipole can always be expressed in terms of the solutions of the following two simpler but more basic problems.

1) The back-scattered signal of an open-circuited dipole: If the dipole is located not too close to the source

[3] Y. Y. Hu, "Back-scattering cross section of a center-loaded cylindrical antenna," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-6, pp. 140–148; January, 1958.

structure, then the higher order effects between the dipole and the source can be neglected. Let $\bar{E}(\bar{r})$ be the field to be measured, and $\bar{I}_0(\bar{r})$ be the current induced on the dipole by $\bar{E}(\bar{r})$, then the electric field intensity produced by $\bar{I}_0(\bar{r})$ can be expressed by the following line integral:

$$\int_{\text{scatterer}} \bar{I}_0(\bar{r}')\cdot\bar{\bar{G}}(\bar{r}', \bar{r})ds'.$$

where $\bar{r}$ and $\bar{r}'$ are the position vectors of the field point and the source point respectively. $\bar{\bar{G}}(\bar{r}', \bar{r})$ is the dyadic Green's function[4] of the dipole with the effect of the source structure neglected. The differential length element ds' indicates that the line integral is to be evaluated with respect to the source coordinates. If s (also s') is considered as a length variable measured along the dipole with $s=0$ at the dipole terminals (these terminals are assumed to be located very close to each other), then there clearly exists a one-to-one correspondence between s and $\bar{r}$. Therefore, $\bar{E}(\bar{r})$ may also be written as $\bar{E}(s)$ for values on the dipole, and the above line integral as

$$\int_{\text{scatterer}} \bar{I}_0(s')\cdot\bar{\bar{G}}(s', s)ds'.$$

These expressions in terms of the variables s and s' should be considered as obtained from a transformation of variables from $\bar{r}$ to s, rather than as a direct substitution of s for $\bar{r}$. Using the boundary condition that the tangential component of the total electric field intensity is zero on the dipole, we then have the following relation over the dipole except at the feed:

$$\bar{E}(s)\cdot\hat{s} + \int_{\text{scatterer}} \bar{I}_0(s')\cdot\bar{\bar{G}}(s', s)\cdot\hat{s}ds' = 0$$

where $\hat{s}$ is a unit tangent vector along the dipole. Using $\hat{s}$, $\bar{I}_0(s)$ may also be written as $I_0(s)\hat{s}$ with $I_0(s)$ as the scalar part of $\bar{I}_0(s)$. At the feed, the value of the above line integral becomes infinity. But if we integrate the left-hand side of the above relation with respect to s over a very small interval from $0-\epsilon$ to $0+\epsilon$, then the first term gives zero and the second term gives a finite value. This finite valued quantity, in fact, is the voltage, V_{10}, induced across the dipole terminals by the electric field intensity to be measured. The above relations satisfy the conditions required for the definition of the Dirac delta function $\delta(s)$; therefore we have

$$V_{10}\delta(s) = \bar{E}(s)\cdot\hat{s} + \int_{\text{scatterer}} \bar{I}_0(s')\cdot\bar{\bar{G}}(s', s)\cdot\hat{s}ds'. \quad (1)$$

It is obvious that the open circuit condition implies

$$I_0(0) = 0. \quad (2)$$

[4] In this dipole case, a scalar Green's function may be used.

For simplicity in notation, a simplified form of integral expression is defined as follows:

$$\int \bar{I}_0\cdot\bar{\bar{G}}\cdot\hat{s} = \int_{\text{scatterer}} \bar{I}_0(s')\cdot\bar{\bar{G}}(s', s)\cdot\hat{s}ds'.$$

Similar notations will be used throughout this paper.

Using the reciprocity theorem, the back-scattered signal V_0 at the source terminals can be expressed in terms of $\bar{I}_0(s)$ as,

$$V_0 = \frac{1}{I}\left\{\int \bar{E}\cdot\bar{I}_0\right\}. \quad (3)$$

2) The signal received from a radiating dipole: If a current I_1 is fed into the dipole terminals, the dipole will radiate. Using the same assumption as given in 1), the following relation is obtained along the dipole:

$$V_{11}\delta(s) = \int \bar{I}_1\cdot\bar{\bar{G}}\cdot\hat{s} \quad (4)$$

where $\bar{I}_1(s) = I_1(s)\hat{s}$ is the radiating current distribution on the dipole, and V_{11} is the voltage across the dipole terminals due to the feeding current I_1. It is obvious we also have

$$I_1(0) = I_1. \quad (5)$$

A simple relation which is useful later can be obtained by multiplying (4) by $I_0(s)$ and integrating over the dipole scatterer,

$$\int\int \bar{I}_1\cdot\bar{\bar{G}}\cdot\bar{I}_0 = 0. \quad (6)$$

The relation can be justified with the use of (2).

Similarly as in 1), the signal V_r received at the source terminals is given by

$$V_r = \frac{1}{I}\left\{\int \bar{E}\cdot\bar{I}_1\right\}. \quad (7)$$

Now in the general case of back-scattered signal due to a loaded dipole, both $\bar{I}_0(s)$ and $\bar{I}_1(s)$ exist simultaneously on the dipole; therefore, the signal V received is the sum of V_0 and V_r.

$$V = V_0 + \frac{I_1}{I}\int \bar{E}\cdot\tilde{I}_1 \quad (8)$$

where $\tilde{I}_1(s)$ is defined by

$$\tilde{I}_1(s) = \frac{\bar{I}_1(s)}{I_1(0)} = \frac{\bar{I}_1(s)}{I_1}. \quad (9)$$

Evidently $\tilde{I}_1(0)$ has unit magnitude.

At the same time, in this loaded dipole case the dipole terminal voltage V_1 is related to the dipole terminal current I_1 by the diode junction impedance Z_L as follows:

$$V_1 = -Z_L I_1. \quad (10)$$

On the other hand, by using the principle of superposition, we have

$$V_1 = V_{10} + V_{11}. \quad (11)$$

Combining (1), (4), (10), and (11), the following relation is obtained:

$$-Z_L I_1 \delta(s) = \bar{E}\cdot\hat{s} + \int \bar{I}_0\cdot\bar{\bar{G}}\cdot\hat{s} + \int \bar{I}_1\cdot\bar{\bar{G}}\cdot\hat{s}. \quad (12)$$

This equation describes the boundary condition along the dipole in the loaded case. Multiplying (12) by $I_1(s)$ and integrating over the dipole, we have

$$-Z_L I_1 = \int \bar{E}\cdot\tilde{I}_1 + I_1 \int\int \tilde{I}_1\cdot\bar{\bar{G}}\cdot\tilde{I}_1. \quad (13)$$

In deriving the above equation, the relations (6) and $\bar{I}_1(s) = I_1\tilde{I}_1(s)$ are used. The double integral in the above equation is known as the input impedance, Z, of the dipole;[3] therefore we have

$$I_1 = -\frac{\int \bar{E}\cdot\tilde{I}_1}{Z + Z_L}. \quad (14)$$

Substituting (14) into (8), V is now given by

$$V = V_0 - \left(\frac{1}{I}\right)\frac{\left\{\int \bar{E}\cdot\tilde{I}_1\right\}^2}{Z + Z_L}. \quad (15)$$

When (15) is applied to the modulated scattering method of measuring $\bar{E}$, it is reasonable to assume that both $\bar{E}$ and $\hat{s}$ are constant along the diode dipole. This assumption is justified by the fact that in measurement the dipole is chosen to be small in comparison with the wavelength. Therefore the factor $\bar{E}\cdot\hat{s}$ can be taken outside of the integral $\int\bar{E}\cdot\tilde{I}_1$, and the remaining factor can be recognized as the effective length l_d of the dipole. $\bar{E}\cdot\hat{s}$ is clearly the component of $\bar{E}$ along the direction of the diode, if it is denoted by E_d, and we then have

$$\int \bar{E}\cdot\tilde{I}_1 = E_d l_d. \quad (16)$$

Finally, the back-scattered signal, V, from a diode dipole is obtained from (15) as

$$V = V_0 - K\frac{E_d^2 l_d^2}{Z + Z_L} \quad (17)$$

with $K = 1/I$. In this equation, the first term is a continuous signal; therefore, it is not detected by the coherent detection system described by Richmond. If the source excitation is kept fixed, K is a constant. The only quantity which is affected by the modulation is the diode junction impedance Z_L appearing in the second term. Therefore the signal due to the second term is modulated and is detected. This shows clearly that the $\bar{E}$ field can be measured by using such a modulated diode as a scatterer.

It should be noted that the modulation used is generally of the on-and-off type. As a result, the diode junction impedance Z_L has essentially two different values—one for the off-period and one for the on-period. From (17), it can be seen that even during the off-period the back-scattered signal due to the second term is not zero, but zero value was used in Richmond's analysis of the coherent detection system. However, with slight modification in his analysis, the following two properties can still be shown:

a) The magnitude of the signal detected in the coherent system is still proportional to the square of the magnitude of the $\bar{E}$ component along the direction of the diode.

b) The phase of the detected signal is now equal to twice the phase angle of $\bar{E}$ plus a constant phase shift. This phase shift depends upon Z and the two different values of Z_L.

II. Back-Scattered Signal From a Diode Loop and $\bar{H}$ Measurement

The scattering loop is formed of two diodes connected as shown in Fig. 2. The loop is assumed to be symmetrical about a line passing through the two diode junctions and also symmetrical about a line passing through the two midpoints on the loop between the two diode junctions. The modulation is applied through a pair of slightly conducting leads to the two diodes in parallel; it modulates both diode junction impedances simultaneously. The applied modulation merely changes the values of these two impedances. As far as the microwave is concerned, the diode loop can be considered simply as an ordinary conducting loop loaded with two impedances due to the two diode junctions.

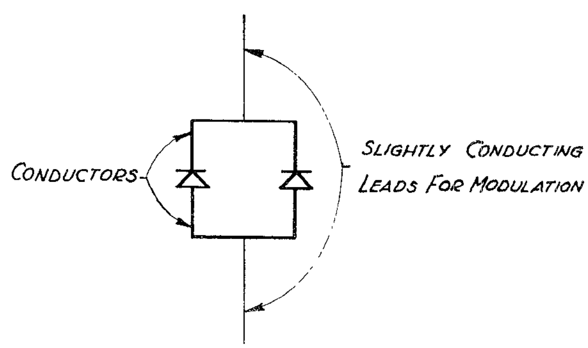

Fig. 2—A diode loop scatterer.

The analysis of a loaded loop is a further extension of that for the loaded dipole, but it is more involved. For simplicity of presentation, some of the obvious assumptions will still be used but not stated explicitly in this analysis. The two terminals of the source and the two pairs of terminals across the two diode junctions are considered as a three-port configuration. The three pairs of terminals of this three-port will be referred to as the source terminals, the loop terminals 1-1, and the loop terminals 2-2. Similarly, as in the case of a loaded dipole, the back-scattered signal at the source terminals due to a loaded loop can be expressed in terms of the following three problems:

1) The back scattered signal of an open-circuited loop. The following equation which corresponds to (1) is obtained:

$$V_{10}\delta(s-s_1)+V_{20}\delta(s-s_2)=\overline{E}\cdot\hat{s}+\int \overline{I}_0\cdot\overline{\overline{G}}\cdot\hat{s} \quad (18)$$

where $\overline{\overline{G}}$ is now the dyadic Green's function for the loop. s is a length variable measured along the loop, but with $s=s_1$ and $s=s_2$ denoting the positions of the two pairs of loop terminals 1-1 and 2-2 respectively. V_{10} and V_{20} are the voltages induced across the loop terminals 1-1 and 2-2. Instead of (2) we now have the following two equations:

$$\overline{I}_0(s_1)=0$$
$$\overline{I}_0(s_2)=0. \quad (19)$$

Eq. (3) still holds, and is repeated here for convenience

$$V_0=\frac{1}{I}\left\{\int \overline{E}\cdot\overline{I}_0\right\}. \quad (20)$$

2) The signal received from a radiating loop excited by a current I_1 at terminals 1-1 but with terminals 2-2 open. A modified form of (4), for this case, is the following:

$$V_{11}\delta(s-s_1)+V_{21}\delta(s-s_2)=\int \overline{I}_1\cdot\overline{\overline{G}}\cdot\hat{s} \quad (21)$$

where V_{11} and V_{21} are the voltages across terminals 1-1 and 2-2 respectively. Evidently we also have

$$I_1(s_1)=I_1$$
$$I_1(s_2)=0 \quad (22)$$

and

$$\iint \overline{I}_1\cdot\overline{\overline{G}}\cdot\overline{I}_0=0. \quad (23)$$

In this case, the signal, V_{r1}, received at the source terminals is given by

$$V_{r1}=\frac{1}{I}\left\{\int \overline{E}\cdot\overline{I}_1\right\}. \quad (24)$$

3) The signal received from a radiating loop excited by a current I_2 at terminals 2-2 but with 1-1 open. It is clear that all the explanations and equations for 2) can be used here simply by interchanging the subscripts 1 and 2.

Now in the general case of a loaded loop, the signal V received is the sum of V_0, V_{r1}, and V_{r2}. It can be written as

$$V=V_0+\frac{I_1}{I}\int \overline{E}\cdot\tilde{I}_1+\frac{I_2}{I}\int \overline{E}\cdot\tilde{I}_2 \quad (25)$$

with

$$\tilde{I}_1(s)=\frac{\overline{I}_1(s)}{I_1} \quad \text{and} \quad \tilde{I}_2(s)=\frac{\overline{I}_2(s)}{I_2}. \quad (26)$$

As in the dipole case, both $\tilde{I}_1(s_1)$ and $\tilde{I}_2(s_2)$ have unit magnitude.

If the diode junction impedances at terminals 1-1 and 2-2 are denoted by Z_{L1} and Z_{L2} respectively, then the voltages V_1 and V_2 across 1-1 and 2-2 can be written as

$$V_1=-Z_{L1}I_1$$
$$V_2=-Z_{L2}I_2. \quad (27)$$

Using the principle of superposition, we have

$$V_1=V_{10}+V_{11}+V_{12}$$
$$V_2=V_{20}+V_{21}+V_{22}. \quad (28)$$

Combining (18), (21), (27), and (28), the following relation is obtained:

$$-Z_{L1}I_1\delta(s-s_1)-Z_{L2}I_Z\delta(s-s_2)$$
$$=\overline{E}\cdot\hat{s}+\int \overline{I}_0\cdot\overline{\overline{G}}\cdot\hat{s}+\int \overline{I}_1\cdot\overline{\overline{G}}\cdot\hat{s}+\int \overline{I}_2\cdot\overline{\overline{G}}\cdot\hat{s}. \quad (29)$$

Multiplying (29) by $\overline{I}_1(s)$ and integrating, we have after using $\overline{I}_1(s)=I_1\tilde{I}_1(s)$,

$$-Z_{L1}I_1=\int \overline{E}\cdot\tilde{I}_1+I_1\iint \tilde{I}_1\cdot\overline{\overline{G}}\cdot\tilde{I}_1$$
$$+I_2\iint \tilde{I}_2\cdot\overline{\overline{G}}\cdot\tilde{I}_1. \quad (30)$$

Similarly, we also have

$$-Z_{L2}I_2=\int \overline{E}\cdot\tilde{I}_2+I_1\iint \tilde{I}_2\cdot\overline{\overline{G}}\cdot\tilde{I}_1$$
$$+I_2\iint \tilde{I}_2\cdot\overline{\overline{G}}\cdot\tilde{I}_2. \quad (31)$$

The four double integrals in (30) and (31) are known as the self and mutual impedances of the loop between the two pairs of terminals. Because of the symmetry conditions assumed for the loop, we may denote the self and mutual impedances simply by Z_s and Z_m. Then we have

$$Z_s=\iint \tilde{I}_1\cdot\overline{\overline{G}}\cdot\tilde{I}_1=\iint \tilde{I}_2\cdot\overline{\overline{G}}\cdot\tilde{I}_2$$
$$Z_m=\iint \tilde{I}_1\cdot\overline{\overline{G}}\cdot\tilde{I}_2=\iint \tilde{I}_2\cdot\overline{\overline{G}}\cdot\tilde{I}_1. \quad (32)$$

Z_s may be described as the loop self impedance across either pair of terminals when the other pair is open, and Z_m, as the loop mutual impedance between the two pairs of terminals of the loop. In terms of Z_s and Z_m, we can then express I_1 and I_2 in (30) and (31) by the following matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = -\begin{bmatrix} Z_s+Z_{L1} & Z_m \\ Z_m & Z_s+Z_{L2} \end{bmatrix}^{-1} \begin{bmatrix} \int \bar{E}\cdot\tilde{I}_1 \\ \int \bar{E}\cdot\tilde{I}_2 \end{bmatrix}. \quad (33)$$

Using this relation, (25) can now be written as

$$V = V_0 - \frac{1}{I}\left[\int \bar{E}\cdot\tilde{I}_1, \int \bar{E}\cdot\tilde{I}_2\right] \cdot \begin{bmatrix} Z_s+Z_{L1} & Z_m \\ Z_m & Z_s+Z_{L2} \end{bmatrix}^{-1} \begin{bmatrix} \int \bar{E}\cdot\tilde{I}_1 \\ \int \bar{E}\cdot\tilde{I}_2 \end{bmatrix}. \quad (34)$$

For the purpose of applying the above relation to the measurement of $\bar{H}$ field, the following two currents—a loop current $\tilde{I}_l(s)$ and a dipole current $\tilde{I}_d(s)$—are defined as

$$\begin{aligned} \tilde{I}_l(s) &= \tilde{I}_1(s) + \tilde{I}_2(s) \\ \tilde{I}_d(s) &= \tilde{I}_1(s) - \tilde{I}_2(s). \end{aligned} \quad (35)$$

The equations in (35) imply

$$\begin{aligned} \tilde{I}_1(s) &= \tfrac{1}{2}(\tilde{I}_l + \tilde{I}_d) \\ \tilde{I}_2(s) &= \tfrac{1}{2}(\tilde{I}_l - \tilde{I}_d). \end{aligned} \quad (36)$$

Substituting (36) into (34) and simplifying, we have

$$V = V_0 - \frac{1}{4ID}\left[\int \bar{E}\cdot\tilde{I}_l, \int \bar{E}\cdot\tilde{I}_d\right] \cdot \begin{bmatrix} 2(Z_s-Z_m)+Z_{L1}+Z_{L2}, & Z_{L2}-Z_{L1} \\ Z_{L2}-Z_{L1}, & 2(Z_s+Z_m)+Z_{L1}+Z_{L2} \end{bmatrix} \begin{bmatrix} \int \bar{E}\cdot\tilde{I}_l \\ \int \bar{E}\cdot\tilde{I}_d \end{bmatrix} \quad (37)$$

where

$$D = (Z_s + Z_{L1})(Z_s + Z_{L2}) - Z_m^2.$$

The symmetric construction of the loop implies that the two different analytical expressions $\tilde{I}_1(s)$ and $\tilde{I}_2(s)$ really represent the same current distribution with respect to their feeding terminals. Both $\tilde{I}_1(s)$ and $\tilde{I}_2(s)$ have unit magnitude at their feeding terminals and are zero at the other pair of terminals. If the loop size is small in comparison with the wavelength, it is reasonable to assume that both $\tilde{I}_1(s)$ and $\tilde{I}_2(s)$ vary linearly from one pair of loop terminals to the other. This implies that $\tilde{I}_l$ is a constant circulating current of unit magnitude flowing around the loop. Using this condition and Stokes' theorem, we have

$$\int \bar{E}\cdot\tilde{I}_l = \int_A \nabla \times \bar{E}\cdot\hat{n}\,dA \quad (38)$$

where $\hat{n}$ is a unit vector normal to the loop plane and A is the area enclosed by the loop. One of the Maxwell's equations reads

$$\nabla \times \bar{E} = -j\omega\mu_0\bar{H}. \quad (39)$$

If it is combined with the assumption that $\bar{H}$ is approximately constant over the loop, we then have

$$\int \bar{E}\cdot\tilde{I}_l \cong -j\omega\mu_0 A H_n \quad (40)$$

where H_n is the component of $\bar{H}$ normal to the loop plane, ω is the microwave angular frequency, and μ_0 is, the permeability of free space.

At the same time, if $\bar{E}$ is also assumed to be approximately constant around the loop, then we may write

$$\int \bar{E}\cdot\tilde{I}_d \cong \bar{E}\cdot\int \tilde{I}_d. \quad (41)$$

The same symmetry conditions imply also that the vector obtained by evaluating the integral $\int\tilde{I}_d$ around the loop will have a direction, $\hat{d}$, which is in the loop plane and perpendicular to the line joining the two diode junctions. This means that the loop with the excitation $\tilde{I}_d$ acts just as a dipole along the direction $\hat{d}$. The magnitude of the integral is defined as the effective length l_d of the loop in that direction. Therefore we again obtain

$$\int \bar{E}\cdot\tilde{I}_d = E_d l_d \quad (42)$$

with E_d as the component of E along the direction $\hat{d}$. In fact, $\hat{d}$ is the direction of the two diodes which form two parallel but opposite sides of the loop.

Substituting (40) and (42) into (37), we have

$$\begin{aligned} V = V_0 - \frac{1}{2ID}\bigg\{&\left(Z_s - Z_m + \frac{Z_{L1}+Z_{L2}}{2}\right)(-j\omega\mu_0 A H_n)^2 \\ &- (Z_{L1} - Z_{L2})(E_d l_d)(-j\omega\mu_0 A H_n) \\ &+ \left(Z_s + Z_m + \frac{Z_{L1}+Z_{L2}}{2}\right)(E_d l_d)^2\bigg\}. \end{aligned} \quad (43)$$

If it is further assumed that $Z_{L1}=Z_{L2}=Z_L$, (43) is then simplified to

$$V = V_0 + K\frac{\omega^2\mu_0^2A^2H_n^2}{Z_s + Z_m + Z_L} - K\frac{E_d^2 l_d^2}{Z_s - Z_m + Z_L} \quad (44)$$

where $K=1/2I$ is a constant, if the source current I is kept fixed. Again Z_L is the only quantity affected by the modulation.

Some properties on the back-scattered signal V, given by (44), are observed as follows:

a) The continuous signal V_0 is not modulated, therefore it is not detected by the coherent detection system.

b) The second term and the third term are modulated, therefore both are subjected to detection.

c) The signal due to the second term is a maximum when $\overline{H}$ is normal to the loop plane, and it is zero when $\overline{H}$ is in the loop plane.

d) The signal due to the third term is a maximum when $\overline{E}$ is parallel to the direction of the diodes, and it is zero when $\overline{E}$ is perpendicular to the direction of the diodes.

These properties justify a procedure for measuring $\overline{H}$. It is clear that the detection system using a modulated diode loop scatterer generally detects a combined signal due to both the $\overline{H}$ field (second term) and the $\overline{E}$ field (third term). In order to eliminate $\overline{E}$ field effect, a modulated diode scatterer is used first to determine the direction of $\overline{E}$. With the direction of $\overline{E}$ known, the $\overline{H}$ field can then be determined by properly orienting the loop. As long as the direction of the diodes is kept perpendicular to $\overline{E}$, the $\overline{E}$ field effect is not detected. If the field to be measured is elliptically polarized, a modified procedure for measuring $\overline{H}$ is given in the Appendix.

The procedure given above applies to the case $Z_{L1} = Z_{L2} = Z_L$, but the same procedure also applies to the case $Z_{L1} \neq Z_{L2}$. This can be seen from (43) which gives the back-scattered signal when $Z_{L1} \neq Z_{L2}$. The relation (43) is considerably more complex than the relation (44), but it still possesses all the essential properties useful for the measurement of the $\overline{H}$ field. In other words, the equal diode junction impedances requirement may be preferred but it is not really necessary.

Appendix

A Procedure for Measuring $\overline{H}$ of an Elliptically-Polarized Field

For an elliptically-polarized field, the $\overline{E}$ vector at a fixed point always lies in a plane[5] which will be called the $\overline{E}$ plane in the following description. It is clear that all the properties of the elliptically-polarized $\overline{E}$ field can be determined by using a diode dipole scatterer, so is the $\overline{E}$ plane. With the $\overline{E}$ plane known, the following procedure or its modification may then be used to determine the $\overline{H}$ field. For the convenience of explanation, a local coordinate system associated with the point at which the $\overline{H}$ field is to be measured will be selected as follows: any two mutually-perpendicular axes in the $\overline{E}$ plane may be used as the x axis and the y axis, and the normal to the $\overline{E}$ plane as the z axis. The measurement of the $\overline{H}$ field is equivalent to the measurement of the three components H_x, H_y, H_z. These can be obtained by locating the geometrical center of the diode loop at the origin of the local coordinate system and then by using the relation (44) four times. First, by setting the diode loop such that the direction of the diodes is perpendicular to the $\overline{E}$ plane and the loop normal is parallel to the x axis, then H_x is obtained. Next, by turning the diode loop about the z axis until the loop normal is parallel to the y axis, and H_y is obtained. By further turning the loop about the y axis until the direction of the diodes is parallel to the x axis, then a combined effect of H_y and E_x is obtained. Eliminating the known result of H_y, E_x is obtained. Finally, by turning the loop about the x axis until the loop lies in the $\overline{E}$ plane, a combined effect of H_z and the same E_x is obtained. By eliminating E_x, H_z is obtained. The above procedure illustrates clearly that the modulated scattering method can be used for measuring the most general type of fields.

[5] Proof of this statement is omitted.

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